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Odds Ratio. An odds ratio is the division of two odds. An odds is formed by two numbers representing two different events (most often event vs. nonevent), such as the odds of winning versus losing, or the odds of voting for the Democratic candidate versus voting for the Republican candidate (or a non-Democratic candidate). As such it is different from a rate, which gives the number of events occurring in a population standardized to a base such as per hundred or per thousand. Odds ratios are useful statistical tools in CATEGORICAL DATA ANALYSIS for studying association or concordance in the data.

G. Udny Yule was notably the first statistician applying odds ratios to measure the association in CONTINGENCY TABLES. Odds ratios and log-odds ratios are common tools for LOGLINEAR MODELING. To demonstrate the application of odds ratios, let us examine in Table 1 some classic data presented by M. Greenwood and G. U. Yule in their 1915 study of the effect of anti-typhoid inoculations (*Proc. R. Soc. Med.* 8, part 2:113-94).

[TABLE 1 ABOUT HERE]

This is a typical 2×2 table with the columns recording the two outcomes and the rows representing the two categories in the exposure variable. More generally, the columns may contain any response or dependent variable and the rows, any explanatory or independent variable; indeed, the rows and columns can simply contain two variables whose association is under investigation. The cells can be indicated by A, B, C, and D (see Table 2).

[TABLE 2 ABOUT HERE]

There are three nonredundant ways of forming odds ratios (*OR*) in a 2×2 table:

$$OR_1 = \frac{A/C}{D/B}, \quad OR_2 = \frac{A/B}{D/C}, \text{ and } OR_3 = \frac{A/B}{C/D}.$$

For studying association between two categorical variables, OR_3, is typically used. This odds ratio is also known as the cross-product ratio because:

$$OR = \frac{A/B}{C/D} = \frac{AD}{BC},$$

where hereafter we ignore the subscript since it is the only odds ratio we examine. Returning to the data on anti-typhoid inoculations, we find the odds ratio or cross-product ratio as

$$OR = \frac{(6{,}759)(272)}{(56)(11{,}396)} = 2.881.$$

The ratio suggests that there is an association between inoculations and typhoid occurrences. The odds of having typhoid prevented are close to three times greater for those who had received inoculations than those who had not. Statistics like these may give some evidence to government agencies for policy making such as implementing vaccination programs.

Note that Yule's *Q* is simply a function of the odds ratio that has been normed to fall between –1 and +1:

$$Q = \frac{OR-1}{OR+1} = \frac{2.881-1}{2.881+1} = 0.485.$$

Because the Q statistic is close to 0.5 (with zero indicating no relationship), it suggests some moderate positive association between inoculations and typhoid prevention.

Often it is convenient to work with (natural-) log-odds ratios, which have two useful properties. Log-odds ratios have zero as the value for indicating no relation between the two variables while that value for odds ratios is one. Zero for no relationship may be intuitively appealing for some researchers. One can also compute asymptotic standard errors for log-odds ratios,

$$\sigma_{\log(OR)} = \sqrt{\frac{1}{A}+\frac{1}{B}+\frac{1}{C}+\frac{1}{D}} = \sqrt{\frac{1}{6{,}759}+\frac{1}{56}+\frac{1}{11{,}396}+\frac{1}{272}} = 0.148,$$

and calculate related Z ratios that follow the standard normal distribution:

$$Z = \frac{\log(OR)}{\sigma_{\log(OR)}} = \frac{0.460}{0.148} = 3.108.$$

The Z statistic indicates a highly significant result (i.e., it is very unlikely that the odds is observed by chance). One can go one step further by constructing confidence intervals for log-odds ratios using the Z obtained (or for odds ratios by going through the anti-log function).

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References

Rudas, T. (1997). *Odds Ratios in the Analysis of Contingency Tables*. Thousand Oaks, CA: Sage.

Table 1: Data on Anti-Typhoid Inoculations

	Typhoid: No	Typhoid: Yes	Total
Inoculation: Yes	6,759	56	6,815
Inoculation: No	11,396	272	11,668
Total	18,155	328	18,483

Table 2: A Two-by-Two Table Layout

	Variable 1, category 1	Variable 1, category 2	Total
Variable 2, category 1	*A*	*B*	*A* + *B*
Variable 2, category 2	*C*	*D*	*C* + *D*
Total	*A* + *C*	*B* + *D*	*A* + *B* + *C* + *D*